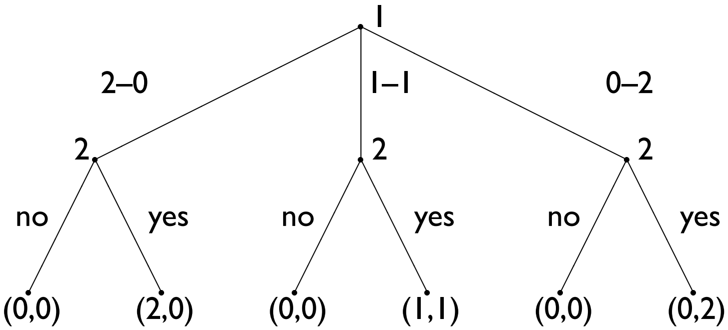
Perfect Information Extensive Form: Strategies, BR, NE

Game Theory Course:
Jackson, Leyton-Brown & Shoham

Example: the sharing game



How many pure strategies does each player have?

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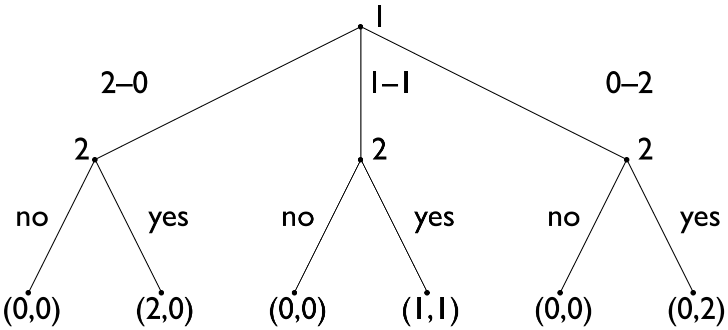

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- player 1: 3

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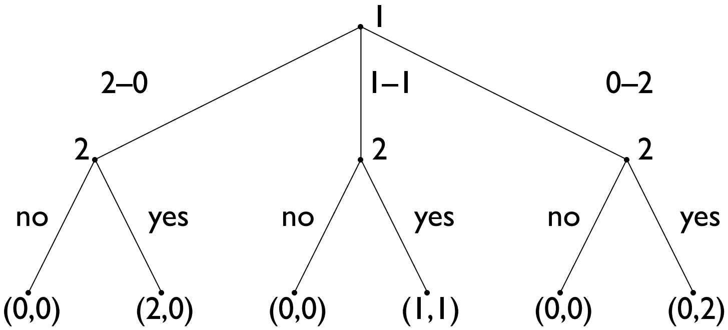


How many pure strategies does each player have?

- player 1: 3
- player 2: 8

Pure Strategies

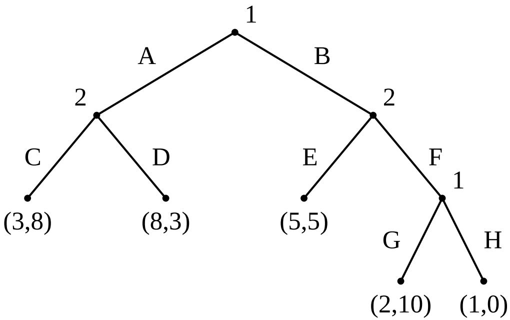
- A pure strategy for a player in a perfect-information game is a complete specification of which action to take at each node belonging to that player.

Definition (pure strategies)

Let $G = (N, A, H, Z, \chi, \rho, \sigma, u)$ be a perfect-information extensive-form game. Then the pure strategies of player i consist of the cross product

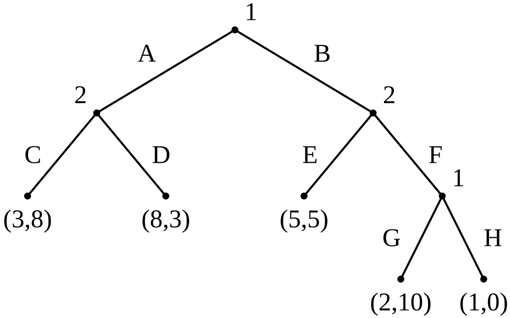
$$\prod_{h \in H, \rho(h)=i} \chi(h)$$

Pure Strategies Example



What are the pure strategies for player 2?

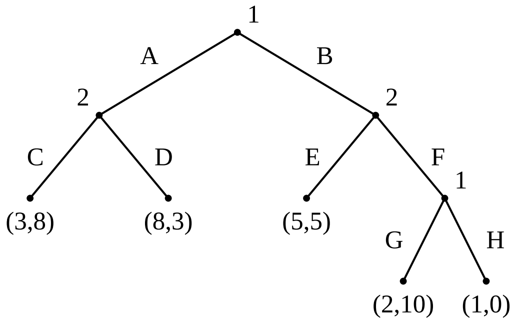
Pure Strategies Example



What are the pure strategies for player 2?

- $S_2 = \{(C, E); (C, F); (D, E); (D, F)\}$

Pure Strategies Example



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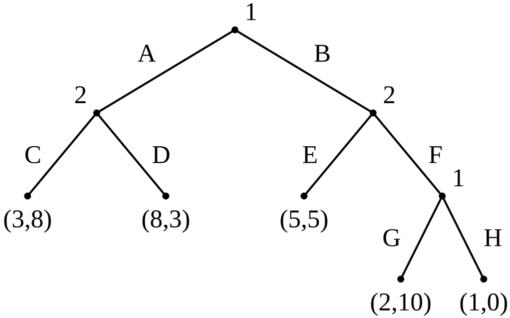
What are the pure strategies for player 1?

Pure Strategies Example



What are the pure strategies for player 2?

- $S_2 = \{(C,E);(C,F);(D,E);(D,F)\}$

What are the pure strategies for player 1?

- $S_1 = \{(B,G);(B,H),(A,G),(A,H)\}$
- This is true even though, conditional on taking A, the choice between G and H will never have to be made

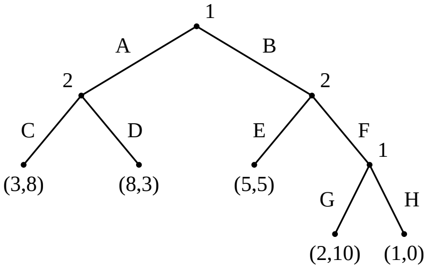
Nash Equilibria

Given our new definition of pure strategy, we are able to reuse our old definitions of:

- mixed strategies
- best response
- Nash equilibrium

Induced Normal Form

- In fact, the connection to the normal form is even tighter
  - we can convert an extensive-form game into normal form



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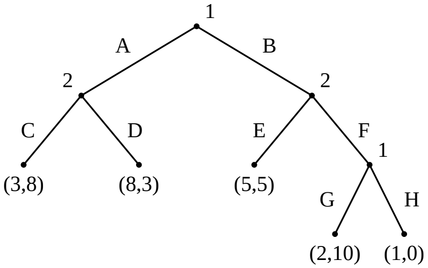

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AG	$3,8$	$3,8$	$8,3$	$8,3$
AH	$3,8$	$3,8$	$8,3$	$8,3$
BG	$5,5$	$2,10$	$5,5$	$2,10$
BH	$5,5$	$1,0$	$5,5$	$1,0$

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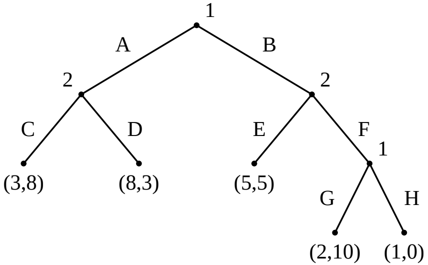

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- this illustrates the lack of compactness of the normal form
  - games aren't always this small
  - even here we write down 16 payoff pairs instead of 5

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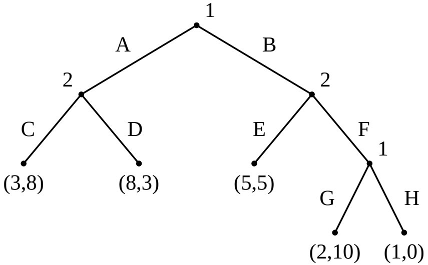

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- we can't always perform the reverse transformation
  - e.g., matching pennies cannot be written as a perfect-information extensive form game

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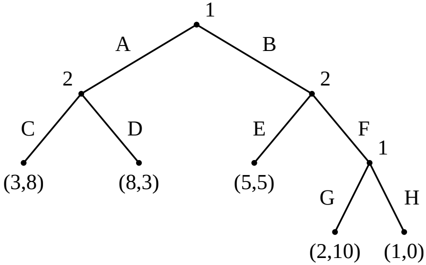

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Theorem

Every perfect information game in extensive form has a PSNE

This is easy to see, since the players move sequentially.

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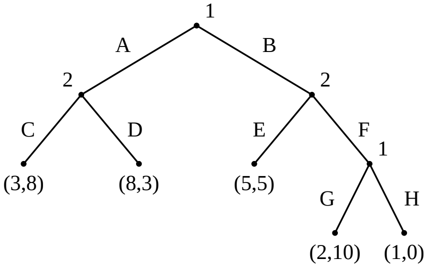

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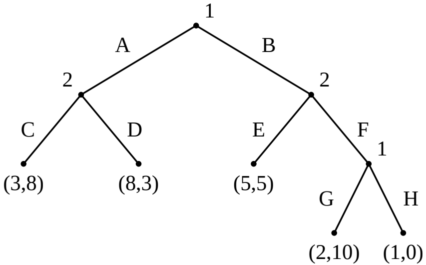

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- What are the (three) pure-strategy equilibria?
  - $(A, G), (C, F)$
  - $(A, H), (C, F)$
  - $(B, H), (C, E)$